1. ELECTRICAL CAPACITANCE — FUNDAMENTALS

DEFINITION + BASIC RELATIONSHIPS

When an electric charge Q is supplied to an isolated conductor, its electrostatic potential V increases proportionally.

$Q = C\,V$

$C = Q / V$

$V = Q / C$

C = constant of proportionality = Electrical Capacitance

Ability of a conductor/system to store electric charge and electrostatic energy per unit potential increase.

UNITS + DIMENSIONAL FORMULA

- SI unit: Farad (F) = 1 Coulomb / Volt = 1 C/V
- 1 μF = 10^{-6} F
- 1 nF = 10^{-9} F
- 1 pF = 10^{-12} F
- Dimensional formula: $[M^{-1} L^{-2} T^{4} A^{2}]$

MEMORY LINK: Q → C → V

CHARGE (Q)
Q = C V

→

CAPACITANCE (C)
Farad

→

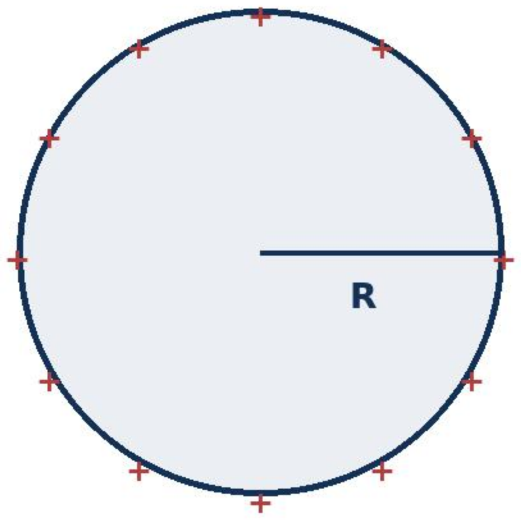
POTENTIAL (V)
V = Q/C

HOT TOPIC: WHY 1 FARAD IS EXTRAORDINARILY LARGE?

For a spherical conductor: $C = 4\pi \varepsilon_0 R$. To have C = 1 F, the source calculates $R = 9 \times 10^{9}$ m = 9,000,000 km. Earth's radius is only 6,400 km = 6.4×10^{6} m, so a 1-F spherical conductor would be about 1,400 times larger than Earth. Hence practical capacitors use μF or pF.

2. CAPACITANCE OF AN ISOLATED SPHERICAL CONDUCTOR

DERIVATION



Step 1 — Potential at surface:
$V = (1 / 4\pi \varepsilon_0) (Q / R)$

Step 2 — Apply $C = Q/V$:
$C = Q / [(1 / 4\pi \varepsilon_0)(Q/R)]$
$C = 4\pi \varepsilon_0 R$

Isolated sphere of radius R carrying total charge Q

$$\boxed{C = 4\pi \varepsilon_0 R}$$

EFFECT OF MEDIUM

If the sphere is placed in a medium of dielectric constant K (relative permittivity ε_r):

$$\boxed{C_{medium} = K\, C_{vacuum}}$$

$$= 4\pi \varepsilon_0 K R$$

CBSE BOARD EXAMPLE 2.1 — EARTH CAPACITANCE

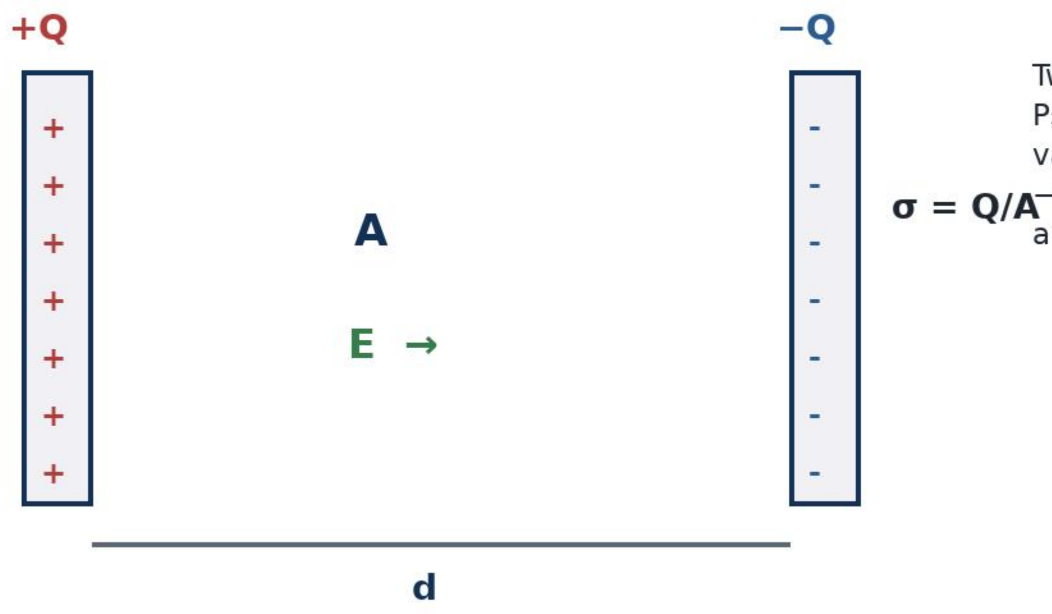
Question: Assuming Earth to be a spherical conductor of radius 6,400 km, calculate its capacitance in microfarads (μF).

- $R = 6{,}400$ km $= 6.4 \times 10^6$ m
- $1/(4\pi \varepsilon_0) = 9 \times 10^9\ N\cdot m^2/C^2$, hence $4\pi \varepsilon_0 = 1/(9 \times 10^9)$ F/m
- Formula: $C = 4\pi \varepsilon_0 R$
- Substitute: $C = (1/9 \times 10^9)(6.4 \times 10^6)$
- $C = 6.4/9000 = 0.0007111$ F
- Convert: $C = 0.0007111 \times 10^6$ μF

FINAL: $C_{Earth} = 711.1$ μF

3. PARALLEL PLATE CAPACITOR — COMPLETE DERIVATION

SETUP + DIAGRAM



Two parallel conducting plates P_1 and P_2, each of area A, separated by d in vacuum. P_1 carries +Q and P_2 carries −Q; surface charge densities are $+\sigma$ and $-\sigma$.

STEP-BY-STEP DERIVATION

1. Electric field between plates

$$E = \sigma/\varepsilon_0 = Q/(A\ \varepsilon_0)$$

In the region between oppositely charged plates, the electric fields add up.

2. Potential difference between plates

$$V = Ed = [Q/(A\ \varepsilon_0)]\ d$$

3. Capacitance

$$C_0 = Q/V = \varepsilon_0\ A/d$$

RESULT + INTERPRETATION

$$C_0 = \varepsilon_0\ A/d$$

- Capacitance increases with plate area A.
- Capacitance decreases when separation d increases.
- $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

PRINCIPLE OF A CAPACITOR

Bringing an earthed neutral conductor near a charged conductor induces opposite charge on the inner side, lowers the potential of the charged plate, and allows it to hold more charge at the same potential.

4. DIELECTRIC SLAB — POLARIZATION + PARTIAL INSERTION

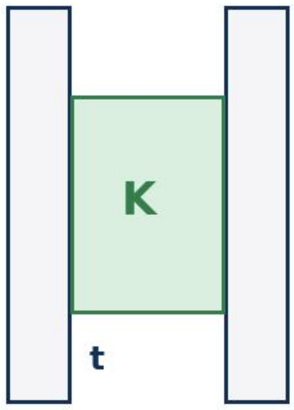
WHAT A DIELECTRIC DOES

External field E_0 → Polarization → Opposing field E_p

Net field decreases: $E = E_0/K$

CASE A — DIELECTRIC SLAB OF THICKNESS t < d

Electric field in air region $(d - t)$: E_0
Electric field inside dielectric slab t: $E = E_0/K$



$V = E_0(d - t) + (E_0/K)t$
$\;\;= E_0 [\, d - t(1 - 1/K) \,]$

$$C = \varepsilon_0 A / [\, d - t(1 - 1/K) \,]$$

SPECIAL CASES + CONDUCTING SLAB

$$t = d \;\rightarrow\; C = K C_0 = K \varepsilon_0 A/d$$

Conducting slab: $C = \varepsilon_0 A/(d - t)$

For a conductor, $K \rightarrow \infty$ and $E = 0$ inside. The effective separation becomes $d - t$.

DIELECTRIC vs CONDUCTOR

DIELECTRIC
- E inside = E_0/K
- K is finite
- Partial slab uses $d - t(1 - 1/K)$

CONDUCTOR
- E inside = 0
- $K \rightarrow \infty$
- Effective thickness = $d - t$

5. BATTERY CONNECTED vs DISCONNECTED

BATTERY CONNECTED

$V = V_0$ (CONSTANT)

Capacitance (C)

Increases: $C = K C_0$

Potential difference (V)

Remains constant: $V = V_0$

Charge (Q)

Increases: $Q = K Q_0$

Electric field (E)

Remains constant: $E = E_0$

Stored energy (U)

Increases: $U = K U_0$

BATTERY DISCONNECTED

$Q = Q_0$ (CONSTANT)

Capacitance (C)

Increases: $C = K C_0$

Potential difference (V)

Decreases: $V = V_0/K$

Charge (Q)

Remains constant: $Q = Q_0$

Electric field (E)

Decreases: $E = E_0/K$

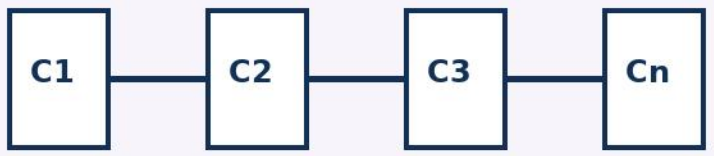
Stored energy (U)

Decreases: $U = U_0/K$

MEMORY TRICK: BATTERY CONNECTED → V is BOSS | DISCONNECTED → Q is BOSS

6. CAPACITORS IN SERIES & PARALLEL

SERIES COMBINATION



Same charge Q on every capacitor

$V = V_1 + V_2 + V_3 + \ldots$

$$1/C_s = 1/C_1 + 1/C_2 + 1/C_3 + \ldots$$

$$C_s = C_1C_2/(C_1 + C_2)$$

VOLTAGE DIVISION

$$V_1 = [C_2/(C_1+C_2)]\, V$$

$$V_2 = [C_1/(C_1+C_2)]\, V$$

Smaller capacitance gets larger voltage in series.

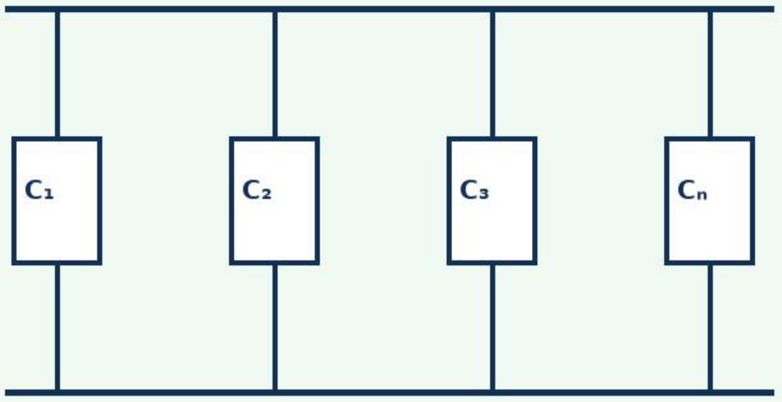
IDENTICAL CAPACITORS

$$C_s = C/n$$

For n identical capacitors of capacitance C:

$$C_p/C_s = n^2$$

PARALLEL COMBINATION



Same voltage V across every capacitor

$Q = Q_1 + Q_2 + Q_3 + \ldots$

$$C_p = C_1 + C_2 + C_3 + \ldots + C_n$$

$$C_p = nC \quad \text{(identical capacitors)}$$

SERIES vs PARALLEL — AT A GLANCE

Charge	Same	Different
Voltage	Different	Same
Equivalent C	Smaller	Larger
Rule	Add reciprocals	Add directly

7. ENERGY STORED IN A CAPACITOR

PHYSICAL MEANING + DERIVATION

Charging a capacitor transfers charge from one plate to another against electrostatic repulsion. The work done is stored as electrostatic potential energy in the electric field.

Let q be the intermediate charge at potential $v = q/C$.

$$\mathbf{dW = v\,dq = (q/C)dq}$$

$$\mathbf{W = \int_0^8 (q/C)dq = Q^2/(2C)}$$

$$\mathbf{U = \tfrac{1}{2}CV^2 = Q^2/(2C) = \tfrac{1}{2}QV}$$

ENERGY DENSITY

Energy density = energy stored per unit volume between the plates.

$$\mathbf{u = \tfrac{1}{2}\,\varepsilon_0\,E^2}$$

SI unit: J/m^3

CHARGE-SHARING: COMMON POTENTIAL

$$\mathbf{V_{common} = (C_1V_1 + C_2V_2)/(C_1 + C_2)}$$

When two charged capacitors are connected in parallel, charges redistribute until a common potential is reached.

ENERGY LOSS ON SHARING CHARGES

$$\mathbf{\Delta U = C_1C_2(V_1 - V_2)^2 / [2(C_1 + C_2)]}$$

The loss of electrostatic energy is dissipated as heat and sparks. It is always positive because $(V_1 - V_2)^2 > 0$.

8. CHARGING & DISCHARGING — RC CONCEPTS

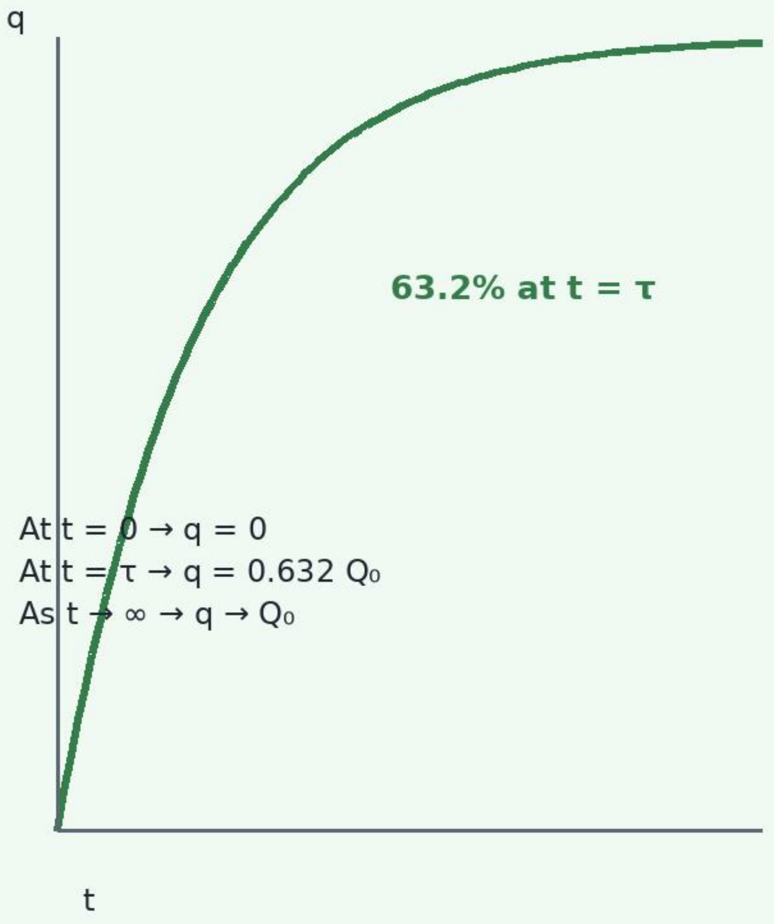
TIME CONSTANT

$$\tau = RC$$

SI unit: seconds. τ represents the time taken for charge to reach 63.2% of maximum during charging, or drop to 36.8% during discharging.

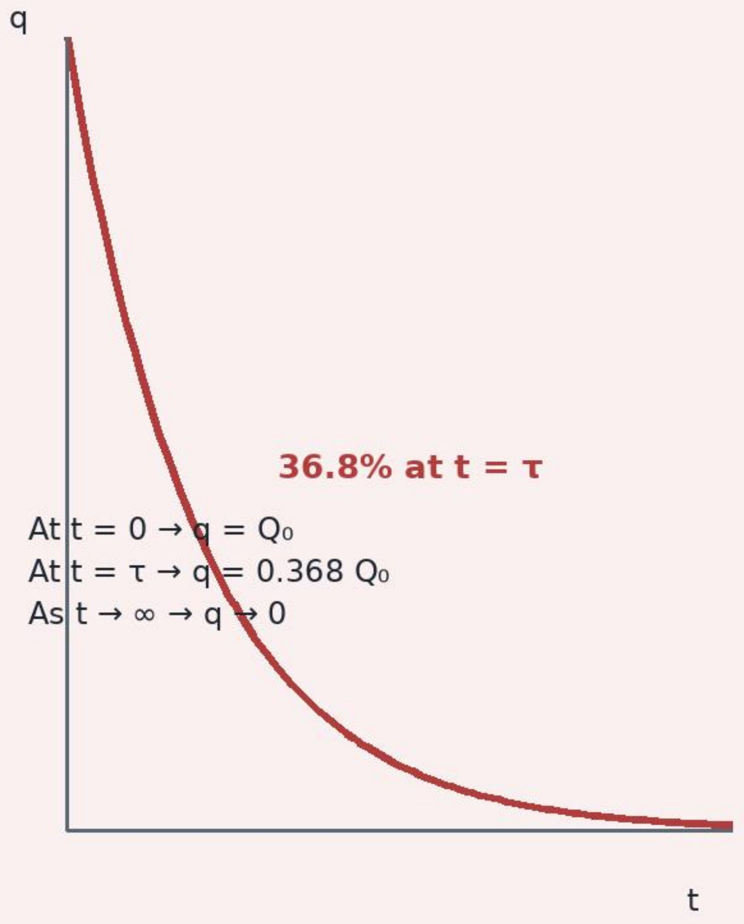
CHARGING

$$q(t) = Q_0(1 - e^{-t/RC})$$



DISCHARGING

$$q(t) = Q_0 e^{-t/RC}$$



9. CBSE PYQ ZONE — ONLY QUESTIONS PRESENT IN SOURCE

PYQ 1 — CBSE 2024, 3 Marks

QUESTION
A parallel plate capacitor of capacitance C is charged to potential V by a battery. The battery is disconnected and a dielectric slab of K = 4 is inserted. Find the ratio of new energy stored to initial energy stored.

$U_0 = \frac{1}{2}CV^2 = Q^2/(2C)$

Battery disconnected → Q constant
$C' = 4C$
$U' = Q^2/(2C') = U_0/4$
FINAL RATIO: $U'/U_0 = 1:4$

PYQ 2 — CBSE 2023, 2 Marks

QUESTION
Two capacitors $C_1 = 3\ \mu F$ and $C_2 = 6\ \mu F$ are connected in series across a 12 V battery. Calculate the voltage across the 3 μF capacitor.

$C_s = (3\times6)/(3+6) = 2\ \mu F$

$Q = C_sV = 24\ \mu C$
Series → same Q
$V_1 = Q/C_1 = 24/3 = 8$ V
FINAL: 8 V

PYQ 3 — CBSE 2022, 3 Marks

QUESTION
Derive an expression for the capacitance of a parallel plate capacitor filled partially with a dielectric slab of thickness t < d.

Use the Case A derivation from Page 4:

$C = \varepsilon_0 A / [d - t(1 - 1/K)]$

PYQ 4 — CBSE 2020, 3 Marks

QUESTION
Two capacitors $C_1 = 2\ \mu F$ and $C_2 = 4\ \mu F$ charged to $V_1 = 100$ V and $V_2 = 50$ V respectively are connected in parallel. Find common potential and energy loss.

$V_{common} = [(2\times100)+(4\times50)]/(2+4) = 66.67$ V

$\Delta U = C_1C_2(V_1-V_2)^2/[2(C_1+C_2)]$
$= 1.67 \times 10^{-3}$ J

10. COMMON EXAM MISTAKES — DON'T LOSE EASY MARKS

Forgetting unit conversions

□ Wrong: substitute μF or pF directly in calculations requiring F.

□ Correct: μF → 10^{-6} F; pF → 10^{-12} F before energy/charge equations.

Confusing series / parallel formulas

□ Wrong: swap capacitor series formula with resistor parallel formula.

□ Correct: Series → add reciprocals; Parallel → add directly.

Misapplying battery condition

□ Wrong: treat V and Q as constant simultaneously.

□ Correct: Battery connected → V constant. Disconnected → Q constant.

Neglecting dielectric slab thickness

□ Wrong: use $K\varepsilon_0 A/d$ for a partial slab.

□ Correct: $C = \varepsilon_0 A/[d - t(1 - 1/K)]$ when $t < d$.

GOLDEN CHECK BEFORE YOU SOLVE

1) Convert units 2) Identify connected/disconnected 3) Decide series/parallel 4) Check dielectric thickness 5) Write the final unit.

11. PRACTICE ZONE — BOARD PATTERN

BASIC
$C = Q/V$
$Q = CV$
$V = Q/C$

SPHERE
$C = 4\pi \varepsilon_0 R$

PARALLEL PLATE
$C_0 = \varepsilon_0 A/d$

DIELECTRIC
$C = K C_0 = K\varepsilon_0 A/d$

PARTIAL SLAB
$C = \varepsilon_0 A/[d - t(1 - 1/K)]$

CONDUCTING SLAB
$C = \varepsilon_0 A/(d - t)$

SERIES
$1/C_s = 1/C_1 + 1/C_2 + \ldots$
Q is same

TWO IN SERIES
$C_s = C_1C_2/(C_1+C_2)$

PARALLEL
$C_p = C_1 + C_2 + \ldots$
V is same

IDENTICAL
$C_s = C/n$
$C_p = nC$
$C_p/C_s = n^2$

ENERGY
$U = \frac{1}{2}CV^2$
$U = Q^2/(2C)$
$U = \frac{1}{2}QV$

ENERGY DENSITY
$u = \frac{1}{2}\varepsilon_0 E^2$
SI: J/m^3

CHARGE SHARING
$V_{common} = (C_1V_1+C_2V_2)/(C_1+C_2)$

ENERGY LOSS
$\Delta U = C_1C_2(V_1-V_2)^2/[2(C_1+C_2)]$

TIME CONSTANT
$\tau = RC$
SI: seconds

CHARGING
$q(t)=Q_0(1-e^{-t/RC})$
63.2% at τ

DISCHARGING
$q(t)=Q_0e^{-t/RC}$
36.8% at τ

MEMORY MAP
ELECTRICAL CAPACITANCE → Definition → Sphere → Parallel Plate → Dielectric → Battery Cases → Series/Parallel → Energy → RC →